

Electroweak Corrections to the Top Quark Decay

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Abstract

We have calculated the one-loop electroweak corrections to the decay $t \rightarrow bW^+$, including the counterterm for the CKM matrix elements V_{tb} . Previous calculations used an incorrect δV_{tb} that led to a gauge dependent amplitude. However, since the contribution stemming from δV_{tb} is small, those calculations only underestimate the width by roughly one part in 10^5 .

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Due to its large mass, $m_t = 174.3 \pm 5.1 \text{ GeV}/c^2$ [1], the top quark, t , decays almost exclusively into a bottom quark, b , and a W -boson. This two-body channel, which is not available to the other quarks, makes the top quark singular. In fact, it is the only known quark where the weak decay takes place before the strong hadronization process. Hence, contrary to the other hadronic weak processes, one can calculate the width for the transition $t \rightarrow bW^+$ without being involved with the non-perturbative aspects of QCD. Because of this advantage this process will be a good testing ground for models beyond the Standard Model (SM). On the other hand, within the SM, the experimental measurement of the decay rate $\Gamma(t \rightarrow bW^+)$ gives a direct measurement of the V_{tb} element of the Cabbibo-Kobayashi-Maskawa (CKM) matrix [2].

Presently, the direct observation of the top quark at the Tevatron [3] implies that V_{tb} is known with a 30% error. The Particle Data Book [1] gives V_{tb} with a smaller error, but using the CKM unitarity conditions. At the CERN LHC, with 10^7 or 10^8 top pairs per year, one expects to extract V_{tb} with an error of the order 10% [4]. It is then desirable to calculate the top width with a few percent precision.

At tree-level the $t \rightarrow bW^+$ width, Γ_0 , is

$$\Gamma_0 = \frac{\alpha}{8 \sin^2 \theta_W} |V_{tb}|^2 \frac{[m_t^2 - (m_W + m_b)^2]^{1/2} [m_t^2 - (m_W - m_b)^2]^{1/2}}{m_t} \left[\frac{m_t^2 + m_b^2}{2m_t^2} + \frac{(m_t^2 - m_b^2)^2}{2m_t^2 m_W^2} - \frac{m_W^2}{m_t^2} \right], \quad (1)$$

where $\alpha = 1/137.03599$ is the fine-structure constant and θ_W is the Weinberg angle ($\cos \theta_W = \frac{m_W}{m_Z}$). The main correction to Γ_0 stems from the one-loop gluon correction to the weak vertex. This $\mathcal{O}(\alpha\alpha_s)$ contribution was first evaluated by Jezabek and Kühn [5], and later confirmed by Denner and Sack(DS) [6] and Eilam et al. [7]. Recently, a similar result was obtained [9] applying the optical theorem to the two-body self-energy of the top quark. At order $(\alpha\alpha_s^2)$ there are two calculations. Czarnecki and Melnikov [10] evaluated the two-loop vertex diagram for $t \rightarrow bW^+$ using the $m_W = 0$ approximation, while Chetyrkin et al. [8] expanded the imaginary part of the three-loop self-energy as a series in q^2/m_t^2 . A recent approach to the same problem by Chinculov and Yao [9] uses a combination of analytical and numerical methods to evaluate the general massive two-loop Feynman diagrams. The electroweak corrections of order α^2 were only evaluated in refs. [6] and [7]. However, as Gambino, Grassi and Madricardo (GGM) [12] have pointed out, in these papers the renormalization of V_{tb} was done in such a way that the final result was gauge dependent. Recently we [13] have considered the renormalization of the CKM matrix, V_{Ij} , in the generic linear R_ξ gauge. We have confirmed that the DS [6] renormalization prescription leads to a gauge dependent amplitude and we have solved the problem introducing a condition to fix δV_{Ij} different from the one proposed by GGM [12]. Despite the fact that DS [6] have used a gauge dependent δV_{Ij} their numerical values for the W partial decay widths are essentially correct. In fact, the δV_{Ij} contribution is negligible. This was confirmed by Kniehl et al. [14] using the GGM prescription. Clearly, it is in the top decay process that a wrong δV_{Ij} would induce the largest difference. In view of this situation we think that it is worthwhile to present, in this note, the correct result for the electroweak one-loop top decay. We will compare our renormalization scheme [13] with the one proposed by GGM [12].

Denoting by p and q the four-momenta of the incoming top quark and the outgoing W^+ , respectively, the tree level decay amplitude T_0 is:

$$T_0 = V_{tb} A_L, \quad (2)$$

with

$$A_L = \frac{g}{\sqrt{2}} \bar{u}(p - q) / \varepsilon \gamma_L u(p), \quad (3)$$

where ε^μ is the polarization vector and, as usual, $\gamma_L = (1 - \gamma_5)/2$. The one-loop amplitude T_1 can be written in terms of four independent form factors, F_L , F_R , G_L and G_R , each one associated with a given Lorentz structure

for the spinors. F_L is associated with A_L and F_R with A_R which is given by eq.(3) replacing γ_L by γ_R . Similarly, G_L and G_R are multiplied by B_L and B_R , respectively, given by:

$$B_{L,R} = \frac{g}{\sqrt{2}} \bar{u}(p-q) \frac{\varepsilon \cdot p}{m_W} \gamma_{L,R} u(p). \quad (4)$$

Besides the form factors, T_1 also depends on the counterterms. The final result is:

$$T_1 = A_L \left[V_{tb} \left(F_L + \frac{\delta g}{g} + \frac{1}{2} \delta Z_W + \frac{1}{2} \delta Z_{tt}^{L*} + \frac{1}{2} \delta Z_{bb}^L \right) + \sum_{I \neq t} \frac{1}{2} \delta Z_{It}^{L*} V_{Ib} + \sum_{j \neq b} V_{tj} \frac{1}{2} \delta Z_{jb}^L + \delta V_{tb} \right] + V_{tb} [A_R F_R + B_L G_L + B_R G_R]. \quad (5)$$

A detailed discussion of the counterterms can be found in our previous work [13] and so there is no need to repeat it here. In particular, we have shown [13] that one obtains a finite and gauge invariant T_1 with the V_{tb} counterterm, δV_{tb} , given by:

$$\delta V_{tb} = -\frac{1}{2} \sum_{I \neq t} \delta Z_{It}^{L*} V_{Ib} - \frac{1}{2} \sum_{j \neq b} V_{tj} \delta Z_{jb}^L - \frac{1}{2} V_{tb} [\delta Z_{tt}^{L*} - \delta Z_{tt[1]}^{L*} + \delta Z_{bb}^L - \delta Z_{bb[1]}^L], \quad (6)$$

where $\delta Z_{II'}^L$ and $\delta Z_{jj'}^L$ are the up and the down left-handed quark wave functions renormalization constants, respectively. A δZ with the subscript [1] means that in its evaluation the CKM matrix was replaced by the identity matrix.

Let us stress that the only difference between our calculation and the previous ones [6, 7] is entirely due to a different choice of δV_{tb} . Unfortunately, the choice made by DS [6] is not physically acceptable. However, as we will see, δV_{tb} gives a rather small contribution. Hence, the numerical result does not show any meaningful change. Perhaps, the best way to discuss the result is to define δ as:

$$\delta = \frac{2Re[T_0 T_1^+]}{|T_0|^2}. \quad (7)$$

This, in turn, means that up to $\mathcal{O}(\alpha^2)$ the decay amplitude can be written as:

$$\Gamma = \Gamma_0 [1 + \delta]. \quad (8)$$

In table 1 we show the different contributions to δ arising from the individual terms of eq.(5). In the calculations the program packages FeynArts [15], FeynCalc [16] and LoopTools [17] were used. Notice that, with our renormalization prescription for δV_{tb} , all contributions from the off-diagonal quark wave-functions renormalization constants are canceled and one simply needs to evaluate $\delta Z_{tt[1]}^{L*}$ and $\delta Z_{bb[1]}^L$. They, together with the other counterterms give a large positive δ (23.66%) which is then reduced to 4.46% with the negative contribution of F_L (-18.75%) and G_R (-0.44%). The other form factors give negligible contributions. It is interesting to see the difference when we follow the CKM renormalization prescription given by GGM [12]. The calculation is slightly more complicated: the off-diagonal terms proportional to δZ_{It}^{L*} and δZ_{jb}^L have to be included; the diagonal terms δZ_{tt}^{L*} and δZ_{bb}^L have to be calculated without the approximation of replacing the CKM matrix by the unit matrix; and finally one ought to add δV_{tb}^G given by:

$$\delta V_{tb}^G = \frac{1}{2} \left[\sum_I \delta Z_{tI}^{L,A} V_{Ib} - \sum_j V_{tj} \delta Z_{jb}^{L,A} \right], \quad (9)$$

where the $\delta\mathcal{Z}_{ij}^{L,A}$ are “special” anti-hermitian wave function renormalization constants fixed in terms of the quark self-energies at $q^2 = 0$, namely,

$$\delta\mathcal{Z}_{ij}^{L,A} = \frac{m_i^2 + m_j^2}{m_i^2 - m_j^2} [\Sigma_{ij}^L(0) + 2\Sigma_{ij}^S(0)]. \quad (10)$$

For the sake of completeness we have also listed in table 1 the numerical values of these additional contributions.

Form Factors and Counterterms	Contributions to $\delta(\%)$
F_L	-18.753
F_R	-2×10^{-3}
G_L	-8×10^{-4}
G_R	-0.445
$\frac{\delta g}{g}$	10.419
$\frac{1}{2}\delta Z_W$	3.193
$\frac{1}{2}\delta Z_{tt}^L$	5.220
$\frac{1}{2}\delta Z_{bb}^L$	4.831
Total	4.46
$\frac{1}{2}\sum_{I \neq t} \delta Z_{It}^{L*} V_{Ib}$	9×10^{-6}
$\frac{1}{2}\sum_{j \neq b} V_{tj} \delta Z_{jb}^L$	-1.8×10^{-3}
$\frac{1}{2}\delta Z_{tt}^L - \frac{1}{2}\delta Z_{tt[1]}^L$	-0.1×10^{-3}
$\frac{1}{2}\delta Z_{bb}^L - \frac{1}{2}\delta Z_{bb[1]}^L$	-5.3×10^{-3}
δV_{tb}^G	6.4×10^{-3}
Total	-0.8×10^{-3}

Table 1: Contributions to δ from the individual terms in eq.(5) evaluated at $m_t = 174.3 \text{ GeV}/c^2$ and $m_H = 114 \text{ GeV}/c^2$. The lower part lists the additional contributions needed if the GGM [12] renormalization prescription is used.

They are all extremely small which means that δ is practically the same in both renormalizations schemes.

Certainly, the uncertainty introduced in the calculation by the error in the top quark mass is far more important. To illustrate this remark and to avoid the need to repeat this calculation in the future we have done it varying m_t in the two-sigma interval around the present experimental mean value. We have found, that within this interval the value of δ can be very well reproduced by the linear fit:

$$\delta = \left[12.7715 - 0.0477 \frac{m_t}{\text{GeV}/c^2} \right] \times 10^{-2}. \quad (11)$$

Figure 1 shows the quality of this fit. Another parameter that enters the calculation is the Higgs mass m_H . In the results given in table 1 and in fig. 1 we have used rather arbitrarily $m_H = 114 \text{ GeV}/c^2$. As it is well known δ depends logarithmically on m_H . Again for $m_t = 174.3 \text{ GeV}/c^2$ and for $100 \text{ GeV}/c^2 \leq m_H \leq 400 \text{ GeV}/c^2$, δ could be fitted with the following expression:

$$\delta = \left[4.4457 + 0.1172 \ln \frac{m_H}{100 \text{ GeV}/c^2} \right] \times 10^{-2}. \quad (12)$$

In figure 2 we show the result and the fitted curve.

We would like to summarize our conclusions as follows:

- i. Using our [13] prescription for the renormalization of the CKM matrix elements we have calculated the electroweak radiative corrections to the decay width $t \rightarrow bW^+$;
- ii. For $m_t = 174.3 \text{ GeV}/c^2$ and $m_H = 114 \text{ GeV}/c^2$, the correction is $\delta = 4.46\%$. This increases the tree level value of Γ from $1.4625 \text{ GeV}/c^2$ to $1.5277 \text{ GeV}/c^2$;
- iii. We have checked that an alternative renormalization prescription advocated by GGM [12] gives a width that differs from ours by less than one part in 10^5 ;
- iv. The contribution to δ stemming from the δV_{tb} counterterm is rather small. It is $7.2 \times 10^{-3}\%$ versus $6.4 \times 10^{-3}\%$ in the GGM [12] scheme, while the old DS [6] δV_{tb} counterterm would have given $6.6 \times 10^{-3}\%$.

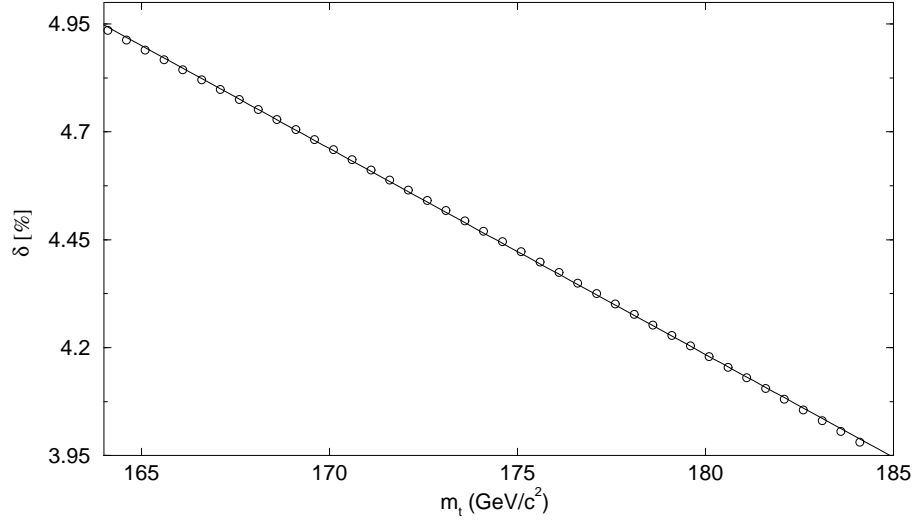


Figure 1: δ as a function of the top mass.

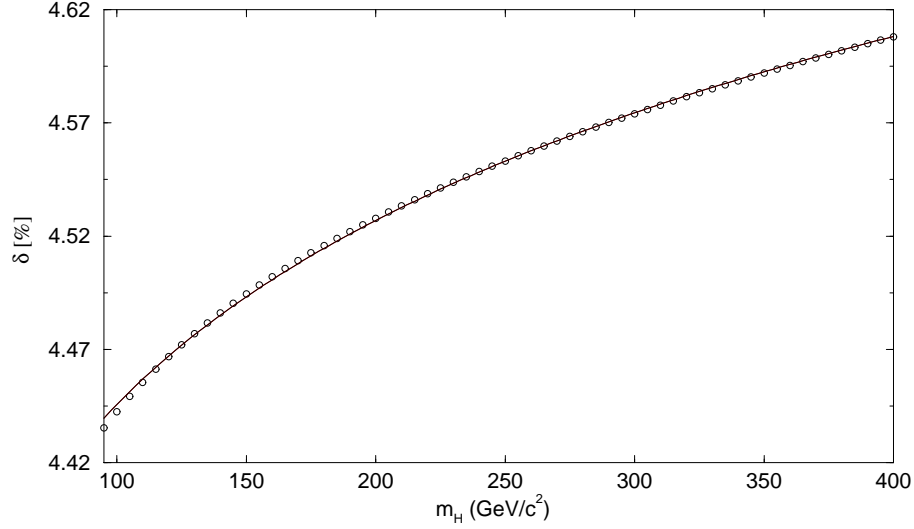


Figure 2: δ as a function of the Higgs mass.

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